

Nucleon Scattering from Very Light Nuclei: Intermediate Energy Expansions for Transition Potentials and Breakup Processes

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Abstract

Optical potentials for elastic p-d scattering and the coupled processes $p+{}^3\text{He} \rightarrow p+{}^3\text{He}$ and $p+{}^3\text{He} \rightarrow d+d$ are derived in the Faddeev-Yakubovsky framework with special emphasis on leading order terms, which are expected to be valid at intermediate energies. In addition, equations for the fragmentations ${}^3\text{He}(p,ppp)n$ and ${}^3\text{He}(p,pp)d$ are derived within the same framework. Again leading order terms for intermediate energies are considered.

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I. INTRODUCTION

Optical potentials have a long tradition in describing scattering of protons and neutrons from composite targets. At intermediate energies their theoretical formulation is often based on approaches like the Watson [1], the KMT formulation [2] or the spectator expansion [3–5]. The fundamental idea from which those approaches start is the grouping of the scattering process for a nucleon hitting a nuclear target into rescatterings of various orders. In lowest order the projectile interacts with one target nucleon, which has a momentum distribution according to a mean field generated by the residual nucleus. In second order the struck target nucleon rescatters from a second target nucleon, which therefore participates actively in the scattering process and is thus no longer ‘hidden’ in the mean field of the target. This more and more complex scenario can be formulated in a general fashion in the so called spectator expansion, but in practice only the lowest order processes have been numerically realized. In the most elaborate case this leads to the so called full-folding model [6–8]. Here, for constructing the optical potential the fully-off-shell nucleon-nucleon (NN) t-matrix is convoluted with the single nucleon density matrix of the target. Antisymmetrization is kept among the target nucleons, but the projectile nucleon is treated as distinguishable and only the antisymmetrization between the active nucleons is kept. This procedure of treating the antisymmetrization is clearly approximative and there might be some flaws in the actual realization as pointed out in the Appendix A. However, this picture turned out to be quite successful in describing proton and neutron scattering from light (^{16}O) to heavy (^{208}Pb) nuclei at intermediate energies using a NN t-matrix derived from a realistic NN force and single nucleon density matrices resulting from nuclear structure calculations [6–8]. Since the structure part faces solving the full A-body problem, approximations have to be made. In addition, it is not the same NN force used in the structure part as the one acting between projectile and target nucleon. This can be considered as an inconsistency. The same theoretical approach has been used to describe elastic scattering of protons from a light nucleus like ^3He at intermediate energies [9,10].

At present only for very light nuclei there is a chance to eliminate this inconsistency concerning the nuclear structure part and treat the scattering process as well as the structure on an equal footing, namely based on the same realistic NN forces. It is the aim of the present study to derive rigorously the optical potential and transition potentials for scattering of nucleons from the two lightest nuclei, the deuteron and ^3He within the framework of the Faddeev-Yakubovsky equations. In this framework, antisymmetry between all nucleons is treated correctly. To exhibit differences as well as similarities to the spectator expansion, we give a brief sketch of the latter scheme in Appendix A.

In Section II we consider the proton-deuteron (p-d) system and in Section III the more complicated p- ^3He system. In the latter case it appears natural to generalize the optical potential for elastic p- ^3He scattering to a potential matrix corresponding to a coupled set of equations describing scattering in both two-body fragmentation channels, p- ^3He and d-d. An additional interest lies in the study of the various breakup processes in p- ^3He scattering into 3 fragments, proton-proton-deuteron (ppd), and into 4 fragments, pppn. For these reactions a wealth of experimental information is available [11,12]. However, these data are conventionally analyzed in PWIA or DWIA only, which does not correspond to a microscopic four particle theory based on NN forces. Three- and four-body fragmentation processes

will be described in Section IV. Throughout the whole article we only consider the strong interaction when describing protons and neglect Coulomb forces. In a practical calculation the Coulomb interaction between the projectile proton and the charge distribution of the nucleus can be included in a straightforward manner at least in the elastic channel [10,13].

In all cases to be discussed, we emphasize intermediate energies where the optical potential expressions and the breakup amplitudes can be expected to simplify. Presently, in the four-body system these simpler expressions appear to be the only ones which can be realized in practical applications. Nevertheless, they carry important information on the reaction process and can serve as testing ground for applying the free NN force in such reactions and as a means to study properties of realistic ^3He wave functions. Appendix B contains the leading terms of the transition potentials between the $p\text{-}^3\text{He}$ and the $d\text{-}d$ channels.

The four-body scattering problem has been formulated in some detail in the context of the Alt-Grassberger-Sandhas (AGS) equations [14,15]. This formulation is basically equivalent to the Faddeev-Yakubovsky one which we are using. In practice however, and especially in the context of deriving optical potentials for $p\text{-}^3\text{He}$ reactions, which is our aim here, it is not necessarily convenient to start from the equations presented in Ref. [14]. In addition, we want to keep our formulation totally independent of assumptions like finite rank approximations to the underlying force. For these reasons we find it adequate to set up the coupled four-body equations our way by regarding the asymptotic behavior of the Faddeev-Yakubovsky equations in configuration space and to introduce at the earliest stage the identity of the nucleons. For the best of our knowledge, this sort of derivation has not been displayed before for scattering processes. For four-nucleon bound states however, this kind of formulation has already been used [16]. We conclude in Section V.

II. ELASTIC PROTON-DEUTERON SCATTERING

It is well known that the operator U for elastic $p\text{-}d$ scattering obeys the AGS equations [14,15]

$$U = PG_0^{-1} + PtG_0U. \quad (2.1)$$

Here G_0 is the free three-nucleon propagator, t the off-shell NN t -operator and P a sum of a cyclic and an anticyclic permutation of 3 particles. This operator equation has to be applied onto the initial channel state

$$|\Phi_{\mathbf{q}_0}\rangle \equiv |\phi_d\rangle |\mathbf{q}_0\rangle \quad (2.2)$$

where $|\phi_d\rangle$ is the deuteron state and $|\mathbf{q}_0\rangle$ the momentum eigenstate of relative motion of the projectile nucleon with respect to the deuteron. Throughout this Section we use the standard Jacobi momenta \mathbf{p} for a two-body subsystem and \mathbf{q} for the ‘spectator’ particle. In addition we work in the isospin formalism and treat the nucleons as identical.

The motion of the three nucleons in intermediate states is now separated into two parts, namely one, where a pair forms a deuteron, and another one where this pair is in the two-body continuum. Denoting by H_0 the operator for the kinetic energy of the three nucleons and by V the potential operator for the NN force, we obtain

$$tG_0 \equiv V(E + i\varepsilon - H_0 - V)^{-1} \equiv VG_b + VG_c \quad (2.3)$$

with

$$G_b \equiv |\phi_d\rangle \frac{1}{E + i\varepsilon - E_d - \frac{3}{4m}q^2} \langle \phi_d| \quad (2.4)$$

and

$$G_c \equiv \int d^3p |\phi_{\mathbf{p}} \rangle^{(+)} \frac{1}{E + i\varepsilon - \frac{p^2}{m} - \frac{3}{4m}q^2} \langle \phi_{\mathbf{p}}^{(+)}|. \quad (2.5)$$

The deuteron binding energy is given by E_d . The set of two-body eigenstates $|\phi_d \rangle$ and $|\phi_{\mathbf{p}} \rangle^{(+)}$ span the two-body Hilbert space and have been used in Eq. (2.4) and Eq. (2.5) to decompose the resolvent operator of Eq. (2.3). In Eqs. (2.4) and (2.5) the denominator is still an operator acting on the spectator motion through the kinetic energy operator $\frac{3}{4m}q^2$.

The driving term in Eq. (2.1) when applied onto the state $|\Phi_{\mathbf{q}_0}\rangle$ given in Eq. (2.2) can also be written as

$$PG_0^{-1}|\Phi_{\mathbf{q}_0} \rangle = PV|\Phi_{\mathbf{q}_0} \rangle, \quad (2.6)$$

where V is acting only within the deuteron.

When inserting the decomposition given in Eq. (2.3) into Eq. (2.1), we obtain

$$U|\Phi_{\mathbf{q}_0} \rangle = PV|\Phi_{\mathbf{q}_0} \rangle + PVG_b U|\Phi_{\mathbf{q}_0} \rangle + PVG_c U|\Phi_{\mathbf{q}_0} \rangle. \quad (2.7)$$

We now define

$$U_c|\Phi_{\mathbf{q}_0} \rangle = PV|\Phi_{\mathbf{q}_0} \rangle + PVG_c U_c|\Phi_{\mathbf{q}_0} \rangle, \quad (2.8)$$

and consequently can rewrite Eq. (2.7) as

$$U|\Phi_{\mathbf{q}_0} \rangle = U_c|\Phi_{\mathbf{q}_0} \rangle + \int d^3q U_c|\Phi_{\mathbf{q}} \rangle g_0(\mathbf{q}) \langle \Phi_{\mathbf{q}}|U|\Phi_{\mathbf{q}_0} \rangle. \quad (2.9)$$

Here Eq. (2.4) has been written out explicitly as

$$G_b = \int d^3q |\phi_d \rangle |\mathbf{q} \rangle \frac{1}{E + i\varepsilon - E_d - \frac{3}{4m}q^2} \langle \phi_d| \langle \mathbf{q}| \equiv \int d^3q |\Phi_{\mathbf{q}} \rangle g_0(\mathbf{q}) \langle \Phi_{\mathbf{q}}|. \quad (2.10)$$

With this preparation Eq. (2.9) can be cast into the closed form of an integral equation by projecting onto the state $\langle \Phi_{\mathbf{q}'}|$

$$\langle \Phi_{\mathbf{q}'}|U|\Phi_{\mathbf{q}_0} \rangle = \langle \Phi_{\mathbf{q}'}|U_c|\Phi_{\mathbf{q}_0} \rangle + \int d^3q \langle \Phi_{\mathbf{q}'}|U_c|\Phi_{\mathbf{q}} \rangle g_0(\mathbf{q}) \langle \Phi_{\mathbf{q}}|U|\Phi_{\mathbf{q}_0} \rangle \quad (2.11)$$

This is the desired integral equation for the elastic p-d scattering amplitude. After a partial wave decomposition it becomes one-dimensional. The driving term

$$\langle \Phi_{\mathbf{q}'}|U_c|\Phi_{\mathbf{q}} \rangle \equiv V_{\mathbf{q}'\mathbf{q}} \quad (2.12)$$

can be interpreted as optical potential $V_{\mathbf{q}'\mathbf{q}}$. The question of interest is, whether at intermediate energies (say above 100 MeV) one can expect that the integral equation, Eq. (2.8), can be solved by iteration and moreover whether the very first few terms will be sufficient. Assuming this to be correct we expand

$$\langle \Phi_{\mathbf{q}'} | U_c | \Phi_{\mathbf{q}} \rangle \approx \langle \Phi_{\mathbf{q}'} | PV | \Phi_{\mathbf{q}} \rangle + \langle \Phi_{\mathbf{q}'} | PV G_c PV | \Phi_{\mathbf{q}} \rangle + \dots \quad (2.13)$$

In order to proceed further, we need to be more specific and identify a two-body subsystem, e.g. we choose the pair (23) to be the deuteron. Then the potential V becomes $V \equiv V_{23}$. The permutation operator P is given by

$$P \equiv P_{12}P_{23} + P_{13}P_{23}, \quad (2.14)$$

and the channel state by

$$|\Phi_{\mathbf{q}}\rangle \equiv |\Phi_{\mathbf{q}}\rangle_1 \equiv |\phi_d\rangle_{23} |\mathbf{q}\rangle_1. \quad (2.15)$$

The first term of Eq. (2.13) is now explicitly given as

$$\langle \Phi_{\mathbf{q}'} | PV | \Phi_{\mathbf{q}} \rangle = {}_1\langle \Phi_{\mathbf{q}'} | V_{31} | \Phi_{\mathbf{q}} \rangle_2 + {}_1\langle \Phi_{\mathbf{q}'} | V_{12} | \Phi_{\mathbf{q}} \rangle_3, \quad (2.16)$$

where

$$\begin{aligned} |\Phi_{\mathbf{q}}\rangle_2 &\equiv P_{12}P_{23}|\Phi_{\mathbf{q}}\rangle_1 \equiv |\phi_d\rangle_{31} |\mathbf{q}\rangle_2 \\ |\Phi_{\mathbf{q}}\rangle_3 &\equiv P_{13}P_{23}|\Phi_{\mathbf{q}}\rangle_1 \equiv |\phi_d\rangle_{12} |\mathbf{q}\rangle_3. \end{aligned} \quad (2.17)$$

With this, one easily finds

$$\langle \Phi_{\mathbf{q}'} | PV | \Phi_{\mathbf{q}} \rangle = {}_1\langle \Phi_{\mathbf{q}'} | V_{13}(1 - P_{13}) | \Phi_{\mathbf{q}} \rangle_2. \quad (2.18)$$

The potential V_{13} , the NN force, should only contribute in two-body states which obey the Pauli principle. Therefore, it has to be represented by antisymmetrized states and thus

$$V_{13}(1 - P_{13}) = 2V_{13}. \quad (2.19)$$

It follows that

$$\langle \Phi_{\mathbf{q}'} | PV | \Phi_{\mathbf{q}} \rangle = 2 {}_1\langle \Phi_{\mathbf{q}'} | V_{13} | \Phi_{\mathbf{q}} \rangle_2. \quad (2.20)$$

This is the well known single particle exchange process where the deuterons in the bracket states are formed out of nucleons (23) and (13), respectively. If the spectator momenta \mathbf{q} and \mathbf{q}' are larger than the typical momenta inside the deuteron this matrix element is small and will not be the leading contribution to the optical potential.

Let us now consider the second term in Eq. (2.13). It is convenient to introduce a t -operator t_c defined as

$$VG_c \equiv t_c G_0. \quad (2.21)$$

From its very definition it is related to the full t -operator by

$$t_c = t - V|\phi_d > g_0 < \phi_d|G_0^{-1} \quad (2.22)$$

and obeys the Lippmann Schwinger equation

$$t_c = V - V|\phi_d > < \phi_d| + t_c G_0 V. \quad (2.23)$$

With the help of Eq. (2.22) and Eq. (2.14) the second term in Eq. (2.13) can be expressed as

$$\begin{aligned} < \Phi_{\mathbf{q}'} | PV G_c PV | \Phi_{\mathbf{q}} > &= < \Phi_{\mathbf{q}'} | P t_c G_0 PV | \Phi_{\mathbf{q}} > \\ &= < \Phi_{\mathbf{q}'} | t_c (31) (1 - P_{13}) G_0 V_{23} | \Phi_{\mathbf{q}} > \\ &+ < \Phi_{\mathbf{q}'} | t_c (21) (1 - P_{12}) G_0 V_{23} | \Phi_{\mathbf{q}} > . \end{aligned} \quad (2.24)$$

If the momentum \mathbf{q} would be on shell, i.e. $|\mathbf{q}| = |\mathbf{q}_0|$, then we would have $G_0 V_{23} |\Phi_{\mathbf{q}}\rangle = |\Phi_{\mathbf{q}}\rangle$ and we would end up with a simple form, namely that nucleon 1 interacts with the constituents of the deuteron (nucleons 2 and 3) through t_c . However, the expression is more complicated. The free propagator is given as

$$G_0 = \frac{1}{E_d - p^2/m} - G_0 (E - E_d - \frac{3}{4m} q^2) \frac{1}{E_d - p^2/m}. \quad (2.25)$$

Therefore,

$$G_0 V_{23} |\Phi_{\mathbf{q}} > = |\Phi_{\mathbf{q}} > - G_0 g_0^{-1} |\Phi_{\mathbf{q}} >, \quad (2.26)$$

so that the second term in Eq. (2.13) is given by

$$\begin{aligned} < \Phi_{\mathbf{q}'} | PV G_c PV | \Phi_{\mathbf{q}} > &= \\ < \Phi_{\mathbf{q}'} | t_c (13) (1 - P_{13}) (1 - G_0 g_0^{-1}) | \Phi_{\mathbf{q}} > &+ < \Phi_{\mathbf{q}'} | t_c (12) (1 - P_{12}) (1 - G_0 g_0^{-1}) | \Phi_{\mathbf{q}} > . \end{aligned} \quad (2.27)$$

This expression can be reduced by replacing $(1 - P_{ij})$ by 2 and noticing that because of the antisymmetry of the deuteron states the two matrix elements are the same. This resulting form for the optical potential has some similarity to the often used ‘ $t\rho$ ’ form as discussed in Appendix A. It will be interesting to perform numerical studies in order to see quantitatively the validity of the truncation in Eq. (2.13) for the optical potential and to compare this expression with the more standard ‘ $t\rho$ ’ form. Such investigations are planned.

III. TWO-BODY FRAGMENTATIONS IN PROTON-³HE SCATTERING

Four-nucleon scattering has not yet been numerically mastered in a rigorous and general manner similar to the scattering of three nucleons upon each other [17]. However, at intermediate energies there is a chance, that specific approximations can be theoretically justified and systematically controlled in a numerical realization. This can open interesting insight into the reaction itself, the use of the free (not modified through the presence of the nuclear medium) NN interaction in such processes as well as into the properties of the three-nucleon bound state. Our approach is a rigorous framework and has been successfully used for the α -particle bound state [18,19].

As is well known the fully antisymmetric scattering state for p-³He scattering, $\Psi^{(+)}$, obeys the homogeneous integral equation

$$\Psi^{(+)} = G_0 \sum_{ij} V_{ij} \Psi^{(+)}, \quad (3.1)$$

where G_0 is the free four-nucleon (4N) propagator and V_{ij} represents the NN force between particles i and j . As a first step one usually decomposes $\Psi^{(+)}$ into 6 Faddeev components

$$\Psi^{(+)} = \sum_{ij} \psi_{ij} \quad (3.2)$$

with

$$\psi_{ij} \equiv G_0 V_{ij} \Psi^{(+)} = G_0 V_{ij} \sum_{kl} \psi_{kl}. \quad (3.3)$$

When introducing the off-shell NN t-matrix t_{ij} , as already defined in Section II, one arrives at 6 coupled Faddeev equations

$$\psi_{ij} = G_0 t_{ij} \sum_{kl \neq ij} \psi_{kl}. \quad (3.4)$$

In the following we immediately make use of the identity of the four particles. This simplifies the notation significantly and it appears not to have been outlaid before in the literature in this particular form. It also serves to clearly define our notation. Without loss of generality we consider the scattering of particle 4 from a target consisting of the subcluster built from particles 1, 2 and 3.

Let us consider the Faddeev component

$$\psi_{12} = G_0 t_{12} (\psi_{23} + \psi_{31} + \psi_{24} + \psi_{14} + \psi_{34}), \quad (3.5)$$

which is then split into 3 Yakubovsky components, namely

$$\psi_1 \equiv G_0 t_{12} (\psi_{23} + \psi_{31}), \quad (3.6)$$

$$P_{34} \psi_1 = -G_0 t_{12} (\psi_{24} + \psi_{14}), \quad (3.7)$$

and

$$\psi_2 \equiv G_0 t_{12} \psi_{34}. \quad (3.8)$$

To arrive at Eq. (3.7) we used the antisymmetry of $\Psi^{(+)}$ and the definition of the Faddeev components as given in Eq. (3.3). This gives

$$\psi_{12} = (1 - P_{34}) \psi_1 + \psi_2. \quad (3.9)$$

With the same reasoning one has

$$\psi_{23} + \psi_{31} = P \psi_{12}, \quad (3.10)$$

where the permutation operator P is given in Eq. (2.14). Thus Eq. (3.6) can be written as

$$\psi_1 = G_0 t_{12} P \psi_{12} = G_0 t_{12} P ((1 - P_{34}) \psi_1 + \psi_2). \quad (3.11)$$

The decisive step to describe p-³He scattering is to sum up all pair forces within the 3-body subcluster of particles 1,2 and 3. In order to achieve this, we rewrite Eq. (3.11) as

$$(1 - G_0 t_{12} P) \psi_1 = G_0 t_{12} P (-P_{34} \psi_1 + \psi_2). \quad (3.12)$$

Now the left hand side alone has a nontrivial solution, which is related to the incoming channel state via

$$(1 - G_0 t_{12} P) \Phi^F = 0 \quad (3.13)$$

with

$$\Phi^F = |\phi^F(123)\rangle |\mathbf{u}\rangle_4. \quad (3.14)$$

Here $|\phi^F\rangle$ denotes the Faddeev component to the target state, namely the 3N bound state $|\phi\rangle$,

$$|\phi\rangle \equiv (1 + P) |\phi^F\rangle. \quad (3.15)$$

The momentum eigenstate of the projectile, $|\mathbf{u}\rangle_4$, is described by an appropriate Jacobi momentum \mathbf{u} . With these definitions we can write Eq. (3.12) as

$$\psi_1 = \Phi^F + (1 - G_0 t_{12} P)^{-1} G_0 t_{12} P (-P_{34} \psi_1 + \psi_2). \quad (3.16)$$

Defining now

$$(1 - G_0 t_{12} P)^{-1} G_0 t_{12} P \equiv G_0 T P, \quad (3.17)$$

which is equivalent to introducing the 3-body operator

$$T = t_{12} + t_{12} P G_0 T, \quad (3.18)$$

one obtains

$$\psi_1 = \Phi^F + G_0 T P (-P_{34} \psi_1 + \psi_2). \quad (3.19)$$

The three-body operator T given in Eq. (3.18) is defined by an off-shell Faddeev equation for the three-body subsystem composed of particles 1,2, and 3, and depends parametrically on the kinetic energy of the fourth particle. The above Eq. (3.19) is the first one of the Yakubovsky-equations.

Let us now consider the second Yakubovsky component from Eq. (3.8). With the help of the permutation operator

$$\tilde{P} \equiv P_{13} P_{24} \quad (3.20)$$

we can write

$$\psi_2 = G_0 t_{12} \tilde{P} \psi_{12} = G_0 t_{12} \tilde{P} ((1 - P_{34}) \psi_1 + \psi_2). \quad (3.21)$$

Next, we sum up the pair forces in the two noninteracting two-body subsystems (12) and (34), representing two deuterons, to infinite order. To do this, we rewrite Eq. (3.21) as

$$(1 - G_0 t \tilde{P}) \psi_2 = G_0 t \tilde{P} (1 - P_{34}) \psi_1 \quad (3.22)$$

and solve for ψ_2 . In the two-deuteron channels there are no ingoing waves, therefore the nontrivial solution to the left hand side alone should not be added and we obtain

$$\psi_2 = (1 - G_0 t \tilde{P})^{-1} G_0 t \tilde{P} (1 - P_{34}) \psi_1 = G_0 \tilde{T} \tilde{P} (1 - P_{34}) \psi_1 \quad (3.23)$$

with

$$\tilde{T} = t_{12} + t_{12} \tilde{P} G_0 \tilde{T}. \quad (3.24)$$

The above equation, Eq. (3.24), defines the T-operator for the two two-particle subclusters, which interact only internally but not with each other. The desired second Yakubovsky equation is given by Eq. (3.23). For the sake of completeness we give also the final expression for the total scattering state

$$\Psi^{(+)} = (1 + P - P_{34} P + \tilde{P}) ((1 - P_{34}) \psi_1 + \psi_2) \quad (3.25)$$

In order to define an optical potential for the scattering of a nucleon from ^3He , we have to separate in Eq. (3.19) the propagation in the ^3He subcluster channel from the propagation, where the three nucleons 1,2, and 3 are in intermediate scattering states. In order to achieve this, we have to reconsider the definition given in Eq. (3.17) in the following way:

$$\begin{aligned} (1 - G_0 t_{12} P)^{-1} G_0 t_{12} P &= (1 - G V_{12} P)^{-1} G V_{12} P \\ &= (E - H_0 - V_{12} (1 + P))^{-1} V_{12} P \\ &\equiv G_0 T P. \end{aligned} \quad (3.26)$$

Here we introduced the resolvent operator

$$G \equiv (E + i\varepsilon - H_0 - V_{12}). \quad (3.27)$$

In the four-body system the convenient ‘odd man out’ notation conventionally used in the three-body system is not applicable and what there appears ‘natural’ as choice of the arbitrarily singled out pair, namely $1 \equiv (23)$, is not being applied to the four-body system. Instead, we have the pair (12) as the ‘first’ pair. Again, the specific choice of the ‘first’ pair is irrelevant and just a matter of convenience.

By the very definition of the target state $|\phi\rangle$ and its Faddeev component $|\phi^F\rangle$ given in Eq. (3.15) one has

$$(E_{He} - h_0(123) - V_{12}(1 + P)) |\phi^F\rangle = 0 \quad (3.28)$$

and

$$< \phi | (E_{He} - h_0(123) - V_{12}(1 + P)) = 0. \quad (3.29)$$

Here the 3N binding energy is given by E_{He} , and $h_0(123)$ stands for the internal kinetic energy of the nucleons 1, 2 and 3. The above two equations indicate that there are different left and right eigenvectors, and thus we obtain

$$(E + i\epsilon - H_0 - V_{12}(1 + P))^{-1} = |\phi^F\rangle g_0 \frac{1}{< \phi | \phi^F >} < \phi | + \text{continuum} , \quad (3.30)$$

where

$$g_0 = \frac{1}{E + i\epsilon - E_{He} - \frac{2}{3m}\mathbf{u}^2} \quad (3.31)$$

is now the single particle propagator in the ^3He subcluster channel, while in Section II it was used for describing the propagation in the deuteron channel. Using Eq. (3.30) we now can cleanly separate this propagation from the rest

$$G_0TP \equiv |\phi^F\rangle g_0 \frac{1}{< \phi | \phi^F >} < \phi | V_{12}P + G_0T^cP. \quad (3.32)$$

Inserted into Eq. (3.19) we arrive at the final expression for the Yakubovsky component ψ_1 :

$$\psi_1 = \Phi^F + |\phi^F\rangle g_0 \frac{1}{< \phi | \phi^F >} < \phi | V_{12}P | (-P_{34}\psi_1 + \psi_2) > + G_0T^cP(-P_{34}\psi_1 + \psi_2). \quad (3.33)$$

Obviously, the second term on the right hand side provides an asymptotically purely outgoing wave carrying the elastic amplitude for p- ^3He scattering

$$M \equiv \frac{1}{< \phi | \phi^F >} < \Phi | V_{12}P(-P_{34}\psi_1 + \psi_2) > . \quad (3.34)$$

Here $\langle \Phi | = \langle \phi | \langle \mathbf{u} |$ is the on-shell channel state. This form is clearly not the standard one and we sketch briefly the link to the more familiar expression. To achieve this, we first rewrite Eq. (3.34) as

$$\begin{aligned} < \phi | \phi^F > M &= < \Phi | V_{12}P(-P_{34}\psi_1 + \psi_2) > \\ &= < \Phi | (E - H_0 - V_{12}) | (-P_{34}\psi_1 + \psi_2) > \\ &= < \Phi | (E - H_0 - V_{12}) | (\psi_{12} - \psi_1) > . \end{aligned} \quad (3.35)$$

The second equality corresponds to Eq. (3.29) for $\langle \Phi |$ as introduced above and the 4N kinetic energy H_0 . The third equality is due to Eq. (3.9). Inserting now the definitions of ψ_{12} and ψ_1 , Eqs. (3.3) and (3.11), we obtain

$$\begin{aligned} < \phi | \phi^F > M &= < \Phi | (E - H_0 - V_{12}) G_0 V_{12} \Psi^{(+)} > - < \Phi | (E - H_0 - V_{12}) G_0 t P G_0 V_{12} | \Psi^{(+)} > \\ &= < \Phi | V_{12} | \Psi^{(+)} > - < \Phi | V_{12} G_0 V_{12} | \Psi^{(+)} > - < \Phi | V_{12} P G_0 V_{12} | \Psi^{(+)} > . \end{aligned} \quad (3.36)$$

Now we use Eq. (3.1) and get together with the explicit definition of the permutation operator as given in Eq. (2.14)

$$\langle \phi | \phi^F \rangle M = \langle \Phi | V_{12} G_0 (V_{14} + V_{24} + V_{34}) | \Psi^{(+)} \rangle. \quad (3.37)$$

Due to the antisymmetry of Φ with respect to particles 1, 2 and 3 and of $(V_{14} + V_{24} + V_{34}) | \Psi^{(+)} \rangle$ this can be rewritten as

$$\begin{aligned} \langle \phi | \phi^F \rangle M &= 1/3 \langle \Phi | (V_{12} + V_{23} + V_{31}) G_0 (V_{14} + V_{24} + V_{34}) | \Psi^{(+)} \rangle \\ &= 1/3 \langle \Phi | V_{14} + V_{24} + V_{34} | \Psi^{(+)} \rangle. \end{aligned} \quad (3.38)$$

In the last equality we took advantage of $\langle \Phi | (V_{12} + V_{23} + V_{31}) G_0 = \langle \Phi |$. Finally we note that $\langle \phi | \phi^F \rangle = 1/3$ for a normalized state $|\phi\rangle$. This concludes our verification that the amplitude M given in Eq. (3.34) is indeed the desired elastic scattering amplitude for p-³He scattering.

Starting from Eq. (3.33) and Eq. (3.23) one can already work out the optical potential formalism for p-³He scattering. We do not want to do this here but rather in addition separate off the propagation of two deuterons in Eq. (3.23). This allows to derive coupled equations for p-³He and d-d scattering. Very likely this will accelerate the convergence of the expansions of the resulting ‘optical’ transition potentials. In order to achieve the separation of the d-d channel we proceed analogously to the separation of the p-³He channel. First we note that in analogy to Eq. (3.26)

$$G_0 \tilde{T} \tilde{P} = (E - H_0 - V_{12}(1 + \tilde{P}))^{-1} V_{12} \tilde{P}. \quad (3.39)$$

Now we need to introduce the analog to ‘Faddeev’ components for the d-d channel, which are somewhat unfamiliar. The two uncorrelated deuterons $\phi_d(12)$ and $\phi_d(34)$ obey the Schrödinger equation

$$(h_0(12) + V_{12} - E_d + h_0(34) + V_{34} - E_d) \phi_d(12) \phi_d(34) = 0, \quad (3.40)$$

where $h_0(ij)$ are internal kinetic energies only. Let us call

$$\phi_{dd} \equiv |\phi_d >_{12} | \phi_d >_{34}. \quad (3.41)$$

In view of Eq. (3.39) we can rewrite Eq. (3.40) as

$$\langle \phi_{dd} | (2E_d - h_0(12) - h_0(34) - V_{12}(1 + \tilde{P})) = 0. \quad (3.42)$$

The integral form of Eqs. (3.42) or (3.40) then reads

$$\phi_{dd} = \frac{1}{2E_d - h_0(12) - h_0(34)} (V_{12} + V_{34}) \phi_{dd}. \quad (3.43)$$

The state ϕ_{dd} can now be decomposed in a Faddeev-like fashion as

$$\phi_{dd} = \phi_{dd}^{F,12} + \phi_{dd}^{F,34}. \quad (3.44)$$

with

$$\phi_{dd}^F \equiv \phi_{dd}^{F,12} \equiv \frac{1}{2E_d - h_0(12) - h_0(34)} V_{12} \phi_{dd} \quad (3.45)$$

and

$$\phi_{dd}^{F,34} = \tilde{P}\phi_{dd}^F. \quad (3.46)$$

It follows that

$$(2E_d - h_0(12) - h_0(34) - V_{12}(1 + \tilde{P}))\phi_{dd}^F = 0. \quad (3.47)$$

Using Eqs. (3.42) and (3.47) we can separate the d-d channel from the rest

$$G_0\tilde{T}\tilde{P} \equiv |\phi_{dd}^F\rangle g_0^{dd} \frac{1}{\langle \phi_{dd}|\phi_{dd}^F\rangle} < \phi_{dd}|V_{12}\tilde{P} + G_0\tilde{T}^c\tilde{P}. \quad (3.48)$$

Here the propagator in the d-d channel is explicitly given by

$$g_0^{dd} \equiv \frac{1}{E - 2E_d + i\epsilon - \frac{1}{2m}\mathbf{v}^2}, \quad (3.49)$$

where \mathbf{v} is a suitable Jacobi momentum for the relative motion of the two deuterons. Insertion of Eq. (3.48) into Eq. (3.23) gives for the second Yakubovsky equation

$$\psi_2 = |\phi_{dd}^F\rangle g_0^{dd} \frac{1}{\langle \phi_{dd}|\phi_{dd}^F\rangle} < \phi_{dd}|V_{12}\tilde{P}(1 - P_{34})\psi_1 + G_0\tilde{T}^c\tilde{P}(1 - P_{34})\psi_1. \quad (3.50)$$

The asymptotic form for the d-d channel follows in a similar fashion as for Eq. (3.33) and the resulting transition amplitude into the d-d channel is given by

$$M_{dd} \equiv \frac{1}{\langle \phi_{dd}|\phi_{dd}^F\rangle} < \Phi_{dd}|V_{12}\tilde{P}(1 - P_{34})\psi_1 >, \quad (3.51)$$

where Φ_{dd} denotes the d-d channel state

$$\Phi_{dd} = |\phi_{dd}\rangle |\mathbf{v}\rangle \quad (3.52)$$

Again, this is not the standard form for the transition amplitude. We want to briefly sketch the relation to the standard form. The basics steps are very similar to the ones leading to the elastic amplitude given in Eq. (3.38). We start from

$$\begin{aligned} \langle \phi_{dd}|\phi_{dd}^F\rangle M_{dd} &= \langle \Phi_{dd}|V_{12}\tilde{P}(1 - P_{34})\psi_1 > \\ &= \langle \Phi_{dd}|(E - H_0 - V_{12})(1 - P_{34})|\psi_1 > \\ &= \langle \Phi_{dd}|(E - H_0 - V_{12})|\psi_{12} - \psi_2 > \\ &= \langle \Phi_{dd}|(E - H_0 - V_{12})G_0V_{12}|\Psi^{(+)} > - \langle \Phi_{dd}|(E - H_0 - V_{12})G_0t\tilde{P}G_0V_{12}|\Psi^{(+)} > \\ &= \langle \Phi_{dd}|V_{12}|\Psi^{(+)} > - \langle \Phi_{dd}|V_{12}G_0V_{12}|\Psi^{(+)} > - \langle \Phi_{dd}|V_{12}\tilde{P}G_0V_{12}|\Psi^{(+)} > \\ &= \langle \Phi_{dd}|V_{12}G_0(V_{13} + V_{14} + V_{23} + V_{24})|\Psi^{(+)} > \\ &= 1/2 \langle \Phi_{dd}|(V_{12} + V_{34})G_0(V_{13} + V_{14} + V_{23} + V_{24})|\Psi^{(+)} > \\ &= 1/2 \langle \Phi_{dd}|(V_{13} + V_{14} + V_{23} + V_{24})|\Psi^{(+)} >. \end{aligned} \quad (3.53)$$

Since $\langle \phi_{dd}|\phi_{dd}^F\rangle = 1/2$ we confirm the standard form.

Now we are ready to derive coupled equations for the p-³He and d-d channels starting from Eqs. (3.33) and (3.50). As a first step we introduce coupled amplitudes for

general Jacobi momenta \mathbf{u} and \mathbf{v} corresponding to the driving terms either in the $p\text{-}^3\text{He}$ or the $d\text{-}d$ channel,

$$\begin{aligned}\psi_{1,\mathbf{u}}^c &= \Phi_{\mathbf{u}}^F + G_0 T^c P(-P_{34}\psi_{1,\mathbf{u}}^c + \psi_{2,\mathbf{u}}^c) \\ \psi_{2,\mathbf{u}}^c &= G_0 \tilde{T}^c \tilde{P}(1 - P_{34})\psi_{1,\mathbf{u}}^c\end{aligned}\quad (3.54)$$

and

$$\begin{aligned}\psi_{1,\mathbf{v}}^d &= G_0 T^c P(-P_{34}\psi_{1,\mathbf{v}}^d + \psi_{2,\mathbf{v}}^d) \\ \psi_{2,\mathbf{v}}^d &= \Phi_{dd,\mathbf{v}}^F + G_0 \tilde{T}^c \tilde{P}(1 - P_{34})\psi_{1,\mathbf{v}}^d\end{aligned}\quad (3.55)$$

Here $\Phi_{dd,\mathbf{v}}^F = \phi_{dd}^F|\mathbf{v}\rangle$ is the ‘Faddeev’ component in the $d\text{-}d$ channel together with the relative momentum eigenstate $|\mathbf{v}\rangle$ of the two deuterons. The second set of equations, Eqs. (3.55), is necessary to derive coupled channel equations. With the above definitions it follows from Eqs. (3.33) and (3.50)

$$\begin{aligned}\psi_1 &= \psi_{1,\mathbf{u}_0}^c + \int d^3u \psi_{1,\mathbf{u}}^c g_0(u) \frac{1}{\langle \phi | \phi^F \rangle} \langle \Phi_{\mathbf{u}} | V_{12} P(-P_{34}\psi_1 + \psi_2) \rangle \\ &+ \int d^3v \psi_{1,\mathbf{v}}^d g_0^{dd}(\mathbf{v}) \frac{1}{\langle \phi_{dd} | \phi_{dd}^F \rangle} \langle \Phi_{\mathbf{v}}^{dd} | V_{12} \tilde{P}(1 - P_{34}) | \psi_1 \rangle\end{aligned}\quad (3.56)$$

and

$$\begin{aligned}\psi_2 &= \psi_{2,\mathbf{u}_0}^c + \int d^3u \psi_{2,\mathbf{u}}^c g_0(\mathbf{u}) \frac{1}{\langle \phi | \phi^F \rangle} \langle \Phi_{\mathbf{u}} | V_{12} P(-P_{34}\psi_1 + \psi_2) \rangle \\ &+ \int d^3v \psi_{2,\mathbf{v}}^d g_0^{dd}(\mathbf{v}) \frac{1}{\langle \phi_{dd} | \phi_{dd}^F \rangle} \langle \Phi_{\mathbf{v}}^{dd} | V_{12} \tilde{P}(1 - P_{34}) | \psi_1 \rangle.\end{aligned}\quad (3.57)$$

Here we denote the initial on-shell relative momentum by \mathbf{u}_0 in contrast to the previous use. The right hand sides contain the transition amplitudes from the initial channel $p\text{-}^3\text{He}$ to the channels $p\text{-}^3\text{He}$ and $d\text{-}d$. Starting from these equations we can easily obtain the following set of coupled equations for half-shell transition amplitudes:

$$\begin{aligned}\langle \Phi_{\mathbf{u}'} | V_{12} P(-P_{34}\psi_1 + \psi_2) \rangle &= \langle \Phi_{\mathbf{u}'} | V_{12} P(-P_{34}\psi_{1,\mathbf{u}_0}^c + \psi_{2,\mathbf{u}_0}^c) \rangle \\ &+ \int d^3u \langle \Phi_{\mathbf{u}'} | V_{12} P(-P_{34}\psi_{1,\mathbf{u}}^c + \psi_{2,\mathbf{u}}^c) \rangle g_0(\mathbf{u}) \frac{1}{\langle \phi | \phi^F \rangle} \langle \Phi_{\mathbf{u}} | V_{12} P(-P_{34}\psi_1 + \psi_2) \rangle \\ &+ \int d^3v \langle \Phi_{\mathbf{u}'} | V_{12} P(-P_{34}\psi_{1,\mathbf{v}}^d + \psi_{2,\mathbf{v}}^d) \rangle g_0^{dd}(\mathbf{v}) \frac{1}{\langle \phi_{dd} | \phi_{dd}^F \rangle} \langle \Phi_{\mathbf{v}}^{dd} | V_{12} \tilde{P}(1 - P_{34}) | \psi_1 \rangle\end{aligned}\quad (3.58)$$

and

$$\begin{aligned}\langle \Phi_{\mathbf{v}'}^{dd} | V_{12} \tilde{P}(1 - P_{34}) | \psi_1 \rangle &= \langle \Phi_{\mathbf{v}'}^{dd} | V_{12} \tilde{P}(1 - P_{34}) | \psi_{1,\mathbf{u}_0}^c \rangle \\ &+ \int d^3u \langle \Phi_{\mathbf{v}'}^{dd} | V_{12} \tilde{P}(1 - P_{34}) | \psi_{1,\mathbf{u}}^c \rangle g_0(\mathbf{u}) \frac{1}{\langle \phi | \phi^F \rangle} \langle \Phi_{\mathbf{u}} | V_{12} P(-P_{34}\psi_1 + \psi_2) \rangle \\ &+ \int d^3v \langle \Phi_{\mathbf{v}'}^{dd} | V_{12} \tilde{P}(1 - P_{34}) | \psi_{1,\mathbf{v}}^d \rangle g_0^{dd}(\mathbf{v}) \frac{1}{\langle \phi_{dd} | \phi_{dd}^F \rangle} \langle \Phi_{\mathbf{v}}^{dd} | V_{12} \tilde{P}(1 - P_{34}) | \psi_1 \rangle\end{aligned}\quad (3.59)$$

Defining transition potentials as

$$\begin{aligned}
V_{\mathbf{u}'\mathbf{u}} &= \langle \Phi_{\mathbf{u}'} | V_{12} P | (-P_{34} \psi_{1,\mathbf{u}}^c + \psi_{2,\mathbf{u}}^c) \rangle \\
V_{\mathbf{v}'\mathbf{v}} &= \langle \Phi_{\mathbf{v}'}^{dd} | V_{12} \tilde{P} (1 - P_{34}) | \psi_{1,\mathbf{v}}^d \rangle \\
V_{\mathbf{u}'\mathbf{v}} &= \langle \Phi_{\mathbf{u}'} | V_{12} P | (-P_{34} \psi_{1,\mathbf{v}}^d + \psi_{2,\mathbf{v}}^d) \rangle \\
V_{\mathbf{v}'\mathbf{u}} &= \langle \Phi_{\mathbf{v}'}^{dd} | V_{12} \tilde{P} (1 - P_{34}) | \psi_{1,\mathbf{u}}^c \rangle,
\end{aligned} \tag{3.60}$$

we can rewrite Eqs.(3.58) and (3.59) in a more transparent form:

$$\begin{aligned}
\langle \Phi_{\mathbf{u}'} | V_{12} P | (-P_{34} \psi_1 + \psi_2) \rangle &= V_{\mathbf{u}'\mathbf{u}_0} \\
&+ \int d^3 u V_{\mathbf{u}'\mathbf{u}} g_0(\mathbf{u}) \frac{1}{\langle \phi | \phi^F \rangle} \langle \Phi_{\mathbf{u}} | V_{12} P | (-P_{34} \psi_1 + \psi_2) \rangle \\
&+ \int d^3 v V_{\mathbf{u}'\mathbf{v}} g_0^{dd}(\mathbf{v}) \frac{1}{\langle \phi_{dd} | \phi_{dd}^F \rangle} \langle \Phi_{\mathbf{v}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) | \psi_1 \rangle
\end{aligned} \tag{3.61}$$

$$\begin{aligned}
\langle \Phi_{\mathbf{v}'}^{dd} | V_{12} \tilde{P} (1 - P_{34}) | \psi_1 \rangle &= V_{\mathbf{v}'\mathbf{u}_0} \\
&+ \int d^3 u V_{\mathbf{v}'\mathbf{u}} g_0(\mathbf{u}) \frac{1}{\langle \phi | \phi^F \rangle} \langle \Phi_{\mathbf{u}} | V_{12} P | (-P_{34} \psi_1 + \psi_2) \rangle \\
&+ \int d^3 v V_{\mathbf{v}'\mathbf{v}} g_0^{dd}(\mathbf{v}) \frac{1}{\langle \phi_{dd} | \phi_{dd}^F \rangle} \langle \Phi_{\mathbf{v}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) | \psi_1 \rangle
\end{aligned} \tag{3.62}$$

These are the coupled integral equations for the two-body fragmentation channels p-³He and d-d and constitute the main result in this study. The solutions lead directly to the elastic amplitude M and the transition amplitude into the d-d channel M_{dd} as given in Eqs. (3.34) and (3.51).

The remaining task is to determine the amplitudes ψ_i^c and ψ_i^d as given in Eqs. (3.54) and (3.55). The idea and hope is that at high enough energies this set of amplitudes can be obtained by iteration and that only the first few terms will contribute. To be explicit let us consider the second order iteration of those equations, which is given by

$$\begin{aligned}
\begin{pmatrix} \psi_{1,\mathbf{u}}^c \\ \psi_{2,\mathbf{u}}^c \end{pmatrix} &= \begin{pmatrix} \Phi_{\mathbf{u}}^F \\ 0 \end{pmatrix} + \begin{pmatrix} G_0 T^c P (-P_{34}) \Phi_{\mathbf{u}}^F \\ G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) \Phi_{\mathbf{u}}^F \end{pmatrix} \\
&+ \begin{pmatrix} G_0 T^c P (-P_{34}) G_0 T^c P (-P_{34}) \Phi_{\mathbf{u}}^F + G_0 T^c P G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) \Phi_{\mathbf{u}}^F \\ G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) G_0 T^c P (-P_{34}) \Phi_{\mathbf{u}}^F \end{pmatrix}
\end{aligned} \tag{3.63}$$

and

$$\begin{aligned}
\begin{pmatrix} \psi_{1,\mathbf{v}}^d \\ \psi_{2,\mathbf{v}}^d \end{pmatrix} &= \begin{pmatrix} 0 \\ \Phi_{dd,\mathbf{v}}^F \end{pmatrix} + \begin{pmatrix} G_0 T^c P \Phi_{dd,\mathbf{v}}^F \\ 0 \end{pmatrix} \\
&+ \begin{pmatrix} G_0 T^c P (-P_{34}) G_0 T^c P \Phi_{dd,\mathbf{v}}^F \\ G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) G_0 T^c P \Phi_{dd,\mathbf{v}}^F \end{pmatrix}
\end{aligned} \tag{3.64}$$

Inserting this result into Eq. (3.60) we find for example the transition potential for p-³He to p-³He scattering in this order of approximation

$$\begin{aligned}
V_{\mathbf{u}'\mathbf{u}} &= \langle \Phi_{\mathbf{u}'} | V_{12} P | (-P_{34}) \Phi_{\mathbf{u}}^F \rangle \\
&+ \langle \Phi_{\mathbf{u}'} | V_{12} P (-P_{34}) G_0 T^c P (-P_{34}) + V_{12} P G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) | \Phi_{\mathbf{u}}^F \rangle \\
&+ \langle \Phi_{\mathbf{u}'} | V_{12} P (-P_{34}) G_0 T^c P (-P_{34}) G_0 T^c P (-P_{34}) + V_{12} P (-P_{34}) G_0 T^c P G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) \\
&+ V_{12} P G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) G_0 T^c P (-P_{34}) | \Phi_{\mathbf{u}}^F \rangle.
\end{aligned} \tag{3.65}$$

The other transition potentials can be obtained in a similar fashion and are given in Appendix B.

In order to evaluate the transition potentials we have to determine T^c and \tilde{T}^c . According to the definitions given in Eqs. (3.32) and (3.48) as well as the defining equations (3.18) and (3.24) for T and \tilde{T} , we obtain after some algebra

$$G_0 T^c = \left[1 - |\phi^F \rangle \frac{1}{\langle \phi | \phi^F \rangle} \langle \phi | \right] G_0 t_{12} + G_0 T^c P G_0 t_{12} \quad (3.66)$$

and

$$G_0 \tilde{T}^c = \left[1 - |\phi_{dd}^F \rangle \frac{1}{\langle \phi_{dd} | \phi_{dd}^F \rangle} \langle \phi_{dd} | \right] G_0 t_{12} + G_0 \tilde{T}^c \tilde{P} G_0 t_{12}. \quad (3.67)$$

As a consequence of the uniqueness of these equations and also from their very definition (the latter one requiring some algebraic manipulations) one can conclude that

$$\langle \phi | G_0 T^c = 0 \quad (3.68)$$

and

$$\langle \phi_{dd} | G_0 \tilde{T}^c = 0 \quad (3.69)$$

This excludes intermediate propagation of ^3He or d-d states in the amplitudes given in Eqs. (3.63) and (3.64). Of course, this should be the case. From solving 3N Faddeev equations at intermediate energies [17], we know that the multiple scattering series converges, except for the ^3He quantum numbers $J^\pi = 1/2^+$. This divergence, however, is due to the very existence of ^3He . Due to the special driving terms in Eqs. (3.66) and (3.67) this divergence is removed. Therefore, there is a good reason to assume that Eqs. (3.66) and (3.67) can be successfully iterated and that very low orders are sufficient. With the abbreviations

$$\Lambda \equiv 1 - |\phi^F \rangle \frac{1}{\langle \phi | \phi^F \rangle} \langle \phi | \quad (3.70)$$

and

$$\Lambda_{dd} \equiv 1 - |\phi_{dd}^F \rangle \frac{1}{\langle \phi_{dd} | \phi_{dd}^F \rangle} \langle \phi_{dd} | \quad (3.71)$$

we consequently approximate $G_0 T^c$ and $G_0 \tilde{T}^c$ to second order in t_{12} as

$$G_0 T^c \rightarrow \Lambda G_0 t_{12} + \Lambda G_0 t_{12} P G_0 t_{12} \quad (3.72)$$

and

$$G_0 \tilde{T}^c \rightarrow \Lambda_{dd} G_0 t_{12} + \Lambda_{dd} G_0 t_{12} \tilde{P} G_0 t_{12}. \quad (3.73)$$

When inserting these expressions into Eq. (3.65) we obtain for the transition potential $V_{\mathbf{u}'\mathbf{u}}$

$$\begin{aligned}
V_{\mathbf{u}'\mathbf{u}} = & \langle \Phi_{\mathbf{u}'} | V_{12} P (-P_{34}) | \Phi_{\mathbf{u}}^F \rangle \\
& + \langle \Phi_{\mathbf{u}'} | V_{12} P P_{34} \Lambda G_0 t_{12} P P_{34} + V_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} (1 - P_{34}) | \Phi_{\mathbf{u}}^F \rangle \\
& + \langle \Phi_{\mathbf{u}'} | V_{12} P P_{34} \Lambda G_0 t_{12} P G_0 t_{12} P P_{34} + V_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} G_0 t_{12} \tilde{P} (1 - P_{34}) | \Phi_{\mathbf{u}}^F \rangle \\
& - \langle \Phi_{\mathbf{u}'} | V_{12} P P_{34} \Lambda G_0 t_{12} P P_{34} \Lambda G_0 t_{12} P P_{34} + V_{12} P P_{34} \Lambda G_0 t_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} (1 - P_{34}) \\
& + V_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} (1 - P_{34}) \Lambda G_0 t_{12} P P_{34} | \Phi_{\mathbf{u}}^F \rangle .
\end{aligned} \tag{3.74}$$

In the parts of $V_{\mathbf{u}'\mathbf{u}}$ which are second order in T^c and \tilde{T}^c we kept only the first order parts of Eqs. (3.72) and (3.73) which are linear in t_{12} .

As one of several possible simplifications in the above expression, we note that $\tilde{P}(1 - P_{34}) = (1 - P_{12})\tilde{P}$ and $(1 - P_{12})$ yields a factor of 2 upon its application to t_{12} . Thus we obtain for the transition potential

$$\begin{aligned}
V_{\mathbf{u}'\mathbf{u}} = & \langle \Phi_{\mathbf{u}'} | V_{12} P (-P_{34}) | \Phi_{\mathbf{u}}^F \rangle \\
& + \langle \Phi_{\mathbf{u}'} | V_{12} P P_{34} \Lambda G_0 t_{12} P P_{34} + 2V_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} | \Phi_{\mathbf{u}}^F \rangle \\
& + \langle \Phi_{\mathbf{u}'} | V_{12} P P_{34} \Lambda G_0 t_{12} P G_0 t_{12} P P_{34} + 2V_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} G_0 t_{12} \tilde{P} | \Phi_{\mathbf{u}}^F \rangle \\
& - \langle \Phi_{\mathbf{u}'} | V_{12} P P_{34} \Lambda G_0 t_{12} P P_{34} \Lambda G_0 t_{12} P P_{34} + 2V_{12} P P_{34} \Lambda G_0 t_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} \\
& + 2V_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} \Lambda G_0 t_{12} P P_{34} | \Phi_{\mathbf{u}}^F \rangle .
\end{aligned} \tag{3.75}$$

The evaluation of this expression requires the know-how of handling all the separate subcluster problems: The ${}^3\text{He}$ bound state and its Faddeev components, the deuteron and the Faddeev component to the two deuteron subcluster problem, the NN t -matrices and above all the (3+1) and (2+2) subcluster kernels $G_0 t_{12} P$ and $G_0 t_{12} \tilde{P}$. In addition there is the P_{34} permutation operator to be considered. All those mathematical structures have been successfully handled in previous work [16,17], and thus the evaluation of this transition potential should pose no additional difficulty, although it is certainly a computational challenge. Similar expressions can be derived for the remaining transition potentials. They are given in Appendix B.

A close inspection of Eq. (3.75) shows that most of the terms will likely not contribute at higher nucleon projectile energies, since they are exchange terms, where the projectile momentum probes bound state wave functions in a region where they are already very small. But there are of course also ‘ $t\rho$ ’-type structures, like part of the fourth term in Eq. (3.75), which will dominate. We leave a more detailed study of Eq. (3.75) to future work. The importance of different terms should become apparent when being supported by numerical realization. Nevertheless we would like to emphasize that the presented approach is systematic. It relies on a mathematically and physically well founded basis. Scattering and bound state structures are treated on the same footing, and antisymmetrization is included fully. The internal validity of our approach can then be checked numerically by adding further terms of the expansions. There will be no free parameters in the calculation, once a certain NN force has been selected.

At a later stage it might be also of interest to study the properties of those transitions potentials with respect to their spin-dependencies and locality versus nonlocality. It would be a surprise if they would have much in common with the often used phenomenological Wood-Saxon type expressions.

IV. THREE- AND FOUR-BODY FRAGMENTATIONS IN PROTON-³HE SCATTERING

As alternative to an algebraic derivation as pursued in the previous section, we would like to arrive at the set of coupled integral equations for the breakup process as given in the Yakubovsky scheme starting from a graphical approach. The resulting equations are exact and can be derived rigorously in an algebraic manner.

The complete breakup process initiated for example by nucleon number 4 striking a ³He target composed of nucleons 1,2, and 3 is given by the infinite sequence of processes depicted in Fig. 1. Here ϕ to the very right of each diagram represents the ³He target ground state and the dashed lines stand for NN interactions. Each diagram has to be read from right to left and each has to start with an interaction between the projectile and one of the constituents of the target. After this initial interaction of the projectile, arbitrary interactions between all four particles have to occur. Clearly, this comprises all possible interactions and intermediate free propagations of the four nucleons. The superscript on the breakup operator $U_0^{(4)}$ indicates that here particle 4 has been singled out as projectile. In order to achieve full antisymmetrization, only the interchange between the projectile and the single target nucleons have to be considered, since the target ground state is assumed to be already antisymmetrized. Thus, the fully antisymmetrized breakup operator U_0 is given by the set of diagrams in Fig. 2. There, the terms representing nucleons 1, 2 or 3 as projectiles enter with a negative sign, since only one transposition is necessary to interchange nucleon 4 with one of them.

For each of the three terms $U_0^{(k)}$, $k = 1, 2, 3$ in Fig. 2 expansions corresponding to $U_0^{(4)}$ as given in Fig. 1 can be written down. The fully antisymmetrized breakup operator U_0 can be decomposed into 6 Faddeev components according to the last pair interaction on the left,

$$U_0 = \sum_{i < j} U_{ij}^0. \quad (4.1)$$

By inspection of the various Born series represented in Fig. 2, one can read off the different Faddeev components. For example, the component U_{34}^0 is given by amplitude

$$U_{34}^0 = V_{34}(\Phi_4 - \Phi_3) + V_{34}G_0 \sum_{i < j} U_{ij}^0. \quad (4.2)$$

Here $|\Phi_4\rangle = |\phi(123)\rangle|\mathbf{u}\rangle_4$ and $|\Phi_3\rangle = P_{34}|\Phi_4\rangle$. As usual one can sum up V_{34} to infinite order into t_{34} :

$$\begin{aligned} U_{34}^0 &= t_{34}(\Phi_4 - \Phi_3) + t_{34}G_0 \sum_{i < j, ij \neq 34} U_{ij}^0 \\ &= t_{34}(\Phi_4 - \Phi_3) + t_{34}G_0(U_{24}^0 + U_{23}^0) \\ &\quad + t_{34}G_0(U_{14}^0 + U_{13}^0) + t_{34}G_0U_{12}^0. \end{aligned} \quad (4.3)$$

In the second equality we already group the terms in a suggestive manner, namely such that the pair indices in the last three terms define two different three-body subclusters, namely 234 and 134, and one 2+2 fragmentation 12-34.

From here on we use the identity of the particles, which simplifies matters considerably and allows to restrict the discussion always to one type of amplitude, either from the partition 3+1 or from the partition 2+2. All remaining amplitudes are obtained by suitable permutations of the particles. Like in the previous section we define a (3+1) Yakubovsky component

$$U_1^0 \equiv t_{34}G_0(U_{24}^0 + U_{23}^0) \quad (4.4)$$

and a (2+2) Yakubovsky component

$$U_2^0 \equiv t_{34}G_0U_{12}^0 \quad (4.5)$$

as two independent amplitudes. Using the identity of the particles, the (3+1) component becomes

$$\begin{aligned} U_1^0 &= t_{34}G_0(-P_{23} - P_{24})U_{34}^0 \\ &= t_{34}G_0(-P_{23} - P_{24}) \left[t_{34}(\Phi_4 - \Phi_3) + U_1^0 - P_{12}U_1^0 + U_2^0 \right]. \end{aligned} \quad (4.6)$$

It is convenient to introduce a permutation operator for particles 2,3, and 4

$$P_{234} = P_{23}P_{34} + P_{24}P_{34}, \quad (4.7)$$

since

$$P_{34}U_{34}^0 = -U_{34}^0. \quad (4.8)$$

Then

$$\begin{aligned} U_1^0 &= t_{34}G_0P_{234}t_{34}(1 - P_{34})\Phi_4 \\ &\quad + t_{34}G_0P_{234}(1 - P_{12})U_1^0 + t_{34}G_0P_{234}U_2^0. \end{aligned} \quad (4.9)$$

Analogous to Eq. (3.26) we sum up all interactions in the 234 subsystem to infinite order and obtain

$$U_1^0 = TG_0P_{234}t_{34}(1 - P_{34})\Phi_4 + TG_0P_{234}(-P_{12}U_1^0 + U_2^0). \quad (4.10)$$

The above equation defines the (off-shell) three-particle operator T as

$$(1 - t_{34}G_0P_{234})^{-1}t_{34}G_0P_{234} \equiv TG_0P_{234}, \quad (4.11)$$

which is equivalent to the integral equation

$$T = t_{34} + t_{34}G_0P_{234}T. \quad (4.12)$$

For this off-shell three-body operator T the same notation is used as in Eq. (3.18), since it represents exactly the same quantity, however it is expressed in different particle numbers. Similar steps as those described above lead to

$$U_2^0 = \tilde{T}G_0\tilde{P}t_{34}(1 - P_{34})\Phi_4 + \tilde{T}G_0\tilde{P}(1 - P_{12})U_1^0, \quad (4.13)$$

where \tilde{P} is given in Eq. (3.20) and

$$\tilde{T} = t_{34} + t_{34}G_0\tilde{P}\tilde{T}. \quad (4.14)$$

Again the same remark as above applies concerning \tilde{T} . The coupled set of Eqs. (4.10) and (4.13) for U_1^0 and U_2^0 are the Yakubovsky equations containing (3+1) and (2+2) subcluster T-operators. Once solved, the Yakubovsky components U_1^0 and U_2^0 are sufficient to generate the full breakup operator U_0 .

Summing all Faddeev components as given in Eq. (4.1) and taking advantage of the identity of the particles yields for the breakup operator

$$U_0 = (1 + P_{234} + \tilde{P} - P_{12}P_{234})U_{34}^0 \quad (4.15)$$

with U_{34}^0 given in Eq. (4.6)

$$U_{34}^0 = t_{34}(1 - P_{34})\Phi_4 + (1 - P_{12})U_1^0 + U_2^0. \quad (4.16)$$

At intermediate energies one can expect that the lowest order terms in the NN t-matrix should be sufficient. When inserting the driving terms of Eqs. (4.12) and (4.14) into the Yakubovsky equations (4.10) and (4.13), we obtain in lowest order

$$\begin{aligned} U_1^0 &\approx t_{34}G_0P_{234}t_{34}(1 - P_{34})\Phi_4 \\ U_2^0 &\approx t_{34}G_0\tilde{P}t_{34}(1 - P_{34})\Phi_4. \end{aligned} \quad (4.17)$$

Thus, in lowest order the breakup operator U_0 is given as

$$U_0 = (1 + P_{234} + \tilde{P} - P_{12}P_{234}) \left[t_{34}(1 - P_{34})\Phi_4 + (1 - P_{12})t_{34}G_0P_{234}t_{34}(1 - P_{34})\Phi_4 + t_{34}G_0\tilde{P}t_{34}(1 - P_{34})\Phi_4 \right]. \quad (4.18)$$

This expression is manifestly antisymmetric in all four particles, since the square bracket is separately antisymmetric in the pairs 12 and 34, as is obvious for the first and second term. Similarly, this can be seen for the third term by noting that $\tilde{P}t_{34}(1 - P_{34}) = t_{12}(1 - P_{12})\tilde{P}$. Now \tilde{P} applied onto Φ_4 yields $-\Phi_2$, which is antisymmetric in 34. The total antisymmetry can then be checked using the permutation operators in front of the square bracket.

As an example for the actual evaluation of the terms in Eq. (4.18) let us consider the term of first order in the NN t-matrix. It is a simple exercise to show that

$$\begin{aligned} &(1 + P_{234} + \tilde{P} - P_{12}P_{234})t_{34}(1 - P_{34})\Phi_4 \\ &= t_{14}(1 - P_{14})\Phi_4 + t_{24}(1 - P_{24})\Phi_4 + t_{34}(1 - P_{34})\Phi_4 \\ &\quad - t_{12}((1 - P_{12})\Phi_2 - t_{13}(1 - P_{13})\Phi_1 - t_{23}((1 - P_{23})\Phi_2). \end{aligned} \quad (4.19)$$

The breakup amplitude results by acting from the left with momentum eigenstates for four free particles. They can be represented by 3 Jacobi momenta suitably chosen according to the pair interacting in the various NN t-matrices. In addition, the various channel states Φ_i , $i = 1, 4$ require adequate choices of Jacobi momenta. For instance Φ_4 is best described by choosing

$$\begin{aligned}
\mathbf{p} &= \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2) \\
\mathbf{q} &= \frac{2}{3}(\mathbf{k}_3 - \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)) \\
\mathbf{r} &= \frac{3}{4} \left(\mathbf{k}_4 - \frac{1}{3}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \right).
\end{aligned} \tag{4.20}$$

Then

$$\langle \mathbf{pqr} | \Phi_4 \rangle = \delta(\mathbf{r} - \mathbf{r}_0) \phi(\mathbf{pq}), \tag{4.21}$$

where \mathbf{r}_0 is the initial projectile momentum (in the zero total momentum frame) and $\phi(\mathbf{pq})$ stands for the 3N bound state. For the sake of simplicity we drop spin and isospin quantum numbers. In order to describe for instance t_{34} it is convenient to introduce a second set of Jacobi momenta, which singles out the relative momentum among particles 3 and 4:

$$\begin{aligned}
\mathbf{p}_1 &= \frac{1}{2}(\mathbf{k}_3 - \mathbf{k}_4) \\
\mathbf{q}_1 &= \frac{2}{3}(\mathbf{k}_2 - \frac{1}{2}(\mathbf{k}_3 + \mathbf{k}_4)) \\
\mathbf{r}_1 &= \frac{3}{4}(\mathbf{k}_1 - \frac{1}{3}(\mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)).
\end{aligned} \tag{4.22}$$

Then one easily derives

$$\begin{aligned}
\langle \mathbf{p}_1 \mathbf{q}_1 \mathbf{r}_1 | \mathbf{pqr} \rangle &= \delta(\mathbf{p} - \frac{2}{3}\mathbf{r}_1 + \frac{1}{2}\mathbf{q}_1) \times \\
&\quad \delta(\mathbf{q} - \frac{2}{3}\mathbf{p}_1 + \frac{2}{3}\mathbf{q}_1 + \frac{4}{9}\mathbf{r}_1) \delta(\mathbf{r} + \mathbf{p}_1 + \frac{1}{2}\mathbf{q}_1 + \frac{1}{3}\mathbf{r}_1) \\
&= \delta(\mathbf{p}_1 + \frac{2}{3}\mathbf{r} - \frac{1}{2}\mathbf{q}) \delta(\mathbf{q}_1 + \frac{2}{3}\mathbf{p} + \frac{2}{3}\mathbf{q} + \frac{4}{9}\mathbf{r}) \times \\
&\quad \delta(\mathbf{r}_1 - \mathbf{p} + \frac{1}{2}\mathbf{q} + \frac{1}{3}\mathbf{r}).
\end{aligned} \tag{4.23}$$

Furthermore, one has

$$\langle \mathbf{p}_1 \mathbf{q}_1 \mathbf{r}_1 | t_{34} | \mathbf{p}'_1 \mathbf{q}'_1 \mathbf{r}'_1 \rangle = t_{34}(\mathbf{p}_1, \mathbf{p}'_1; E - \frac{3}{4m}q_1^2 - \frac{2}{3m}r_1^2) \delta(\mathbf{q}_1 - \mathbf{q}'_1) \delta(\mathbf{r}_1 - \mathbf{r}'_1). \tag{4.24}$$

Therefore, using Eqs. (4.21) and (4.24) one easily finds

$$\begin{aligned}
\langle \mathbf{p}_1 \mathbf{q}_1 \mathbf{r}_1 | t_{34} (1 - P_{34}) \Phi_4 \rangle &= \\
2 t_{34}(\mathbf{p}_1, -\mathbf{r}_0 - \frac{1}{2}\mathbf{q}_1 - \frac{1}{3}\mathbf{r}_1; E - \frac{3}{4m}q_1^2 - \frac{2}{3m}r_1^2) \phi\left(\frac{2}{3}\mathbf{r}_1 - \frac{1}{2}\mathbf{q}_1, -\frac{2}{3}\mathbf{r}_0 - \mathbf{q}_1 - \frac{2}{3}\mathbf{r}_1\right).
\end{aligned} \tag{4.25}$$

This term is maximal under the condition, that both arguments in ϕ are zero. This leads to

$$\begin{aligned}
\mathbf{q}_1 &= -\frac{4}{9}\mathbf{r}_0 \\
\mathbf{r}_1 &= -\frac{1}{3}\mathbf{r}_0
\end{aligned} \tag{4.26}$$

Noting that $\mathbf{r}_0 = \frac{3}{4}\mathbf{k}_{lab}$, where \mathbf{k}_{lab} is the projectile momentum in the laboratory system, one easily deduces by using Eq. (4.22), that the conditions Eq. (4.26) are equivalent to $\mathbf{k}_1 = \mathbf{k}_2 = 0$. The condition of zero momenta for the spectator nucleons is usually called quasifree scattering.

Next we consider the arguments of the NN t-matrix t_{34} in Eq. (4.25). Using

$$E = E_{He} + \frac{2}{3m}r_0^2 = \frac{p_1^2}{m} + \frac{3}{4m}q_1^2 + \frac{2}{3m}r_1^2, \quad (4.27)$$

where E_{He} is the binding energy of ${}^3\text{He}$, one finds under the condition given in Eq. (4.26)

$$t_{34}(\mathbf{p}_1, -\frac{2}{3}\mathbf{r}_0; E_{He} + \frac{1}{m}(\frac{2}{3}\mathbf{r}_0)^2). \quad (4.28)$$

Therefore t_{34} is on the energy shell except for the negative binding correction E_{He} of the ${}^3\text{He}$ target.

Going away from the quasielastic peak one can probe the off-shell NN t-matrix and the ${}^3\text{He}$ target wave function. At the same time however, the other terms in Eq. (4.19) will also contribute. Each term in Eq. (4.19) expressed in the adequate Jacobi momenta will have a form similar to Eq. (4.25). Corresponding spectator momenta set to zero have the effect, that one term will dominate. The other ones are suppressed, since the arguments of ϕ then differ from zero and the t-matrices are off-shell. Numerical studies are required to learn about interferences among the six terms if one is away from the peak conditions. One can expect to extract interesting information on the ground state wave function ϕ of ${}^3\text{He}$ and the off-shell NN t-matrix. Both are nowadays readily accessible for realistic NN forces and can therefore be tested.

The next step is to evaluate the terms second order in t_{34} as given in Eq. (4.18). This is more complicated, since it involves the free propagator, which together with the permutation operators leads to logarithmic singularities. However, their treatment is known from the three-body system [17], and thus these terms can also be numerically determined. We expect that with increasing energy the multiple scattering series should terminate quickly and therefore the reaction as well as properties of ${}^3\text{He}$ can be tested systematically and in a rigorous way. This will be left to future work.

A final step is the formulation of the transition to the three-body fragmentation channels d+p+p. Again we start by choosing particle 4 as projectile and display the first terms of the infinite series of processes in Fig. 3. Here one has to note that the last interaction to the left cannot take place between particles 1 and 2, since V_{12} is already taken into account in the deuteron ground state wave function $\phi_d(12)$. We recognize the same series of processes as in Fig. 1, except that the last interaction V_{12} is excluded. The antisymmetrization in the initial state leads to the symmetrized transition operator

$$U_d = U_d^{(4)} - \sum_{i=1}^3 U_d^{(i)}, \quad (4.29)$$

where $U_d^{(i)}$ refers to the projectile i . For $i = 1, 2$ there occurs just one term with V_{12} , namely $V_{12}\phi_i$, $i = 1, 2$. Thus we have

$$U_d = -V_{12}(\phi_1 + \phi_2) + \sum_{i<j, ij \neq 12} U_{ij}^0. \quad (4.30)$$

For the on-shell process the potential V_{12} can be replaced by G_0^{-1} . Finally, permutation operators can be introduced leading to

$$U_d = -G_0^{-1}(P_{14} + P_{24})\Phi_4 + (1 + P - P_{12}P)U_{34}^0. \quad (4.31)$$

Here U_{34}^0 is given in terms of the two Yakubovsky amplitudes U_1^0 and U_2^0 of Eq. (4.16). Again the lowest order terms in Eq. (4.17) are expected to be sufficient at intermediate energies. This remains to be verified in a numerical study.

V. SUMMARY AND OUTLOOK

Elastic proton-deuteron scattering is formulated within the framework of Faddeev equations to achieve the form of a one-body equation with an optical potential as driving term. The exact equation defining this potential is assumed to be solvable at intermediate energies by a low order expansion in the NN t-matrix. This remains to be verified numerically. The presented formulation is a systematic expansion in two-nucleon t-matrices and treats the antisymmetry among all three particles exactly. A standard approach to derive microscopic optical potentials at intermediate energies is the ‘spectator expansion’ of multiple scattering theory. We apply this method in the case of p-deuteron scattering in order to illustrate similarities and differences to the treatment within the Faddeev scheme.

In the four-nucleon system we formulate an exact set of coupled equations for p- ^3He and d-d scattering within the framework of the Yakubovsky equations. The transition potentials between those two channels are again approximated at intermediate energies in a low order expansion in the NN t-matrix. This expansion is systematic and antisymmetrization among all four particles is treated exactly. The numerical realization as well as the internal check of convergence with respect to higher order terms should provide interesting insight into the reaction mechanism and should lay a firm ground to test various physical assumptions, like the application of free NN forces in such a reaction as well as the properties of the ^3He wave function as resulting from solving the Faddeev equations based again on NN forces.

We also derived exact equations for the p- ^3He induced three- and four-body fragmentation processes. Here we again concentrate on the lowest order terms in a NN t-matrix expansion, which we expect to be valid at intermediate energies. The present work lies a formal ground for numerical investigations, which are planned. The expectation is that at least at intermediate energies the four-nucleon scattering problem can be numerically controlled in a reliable manner, using the outlaid systematic t-matrix expansion. At lower energies such an expansion fails, as is already known from the 3N system [17]. In the low energy regime an exact solution of the four-nucleon Yakubovsky equations appears not to be feasible at the present time. Since the proposed scheme does not contain any adjustable parameters once a realistic NN force has been chosen, its numerical realization and comparison to experiment should provide interesting insight into our understanding of such reaction processes and the properties of the ^3He wave function.

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APPENDIX A: THE SPECTATOR EXPANSION FOR P-D SCATTERING

In this Appendix, the most simple system for nucleon-nucleus scattering, the p-d system, is used to explain the steps involved in the spectator expansion of multiple scattering theory, on which one model for the nucleon-nucleus optical potential is based [5]. Since the p-d system is the simplest system for nucleon-nucleus scattering all steps can be clearly carried out and the result can be compared to the exact Faddeev framework of Section II. We especially want to emphasize the treatment of the antisymmetrization and show the inherent limitation in spectator expansion. For distinguishable particles the spectator expansion is carried out for three particles in Ref. [20].

In the general derivation of the optical potential for scattering of a nucleon from a composite nucleus, the protons and neutrons are treated as distinguishable particles. In this Appendix we will follow this general practice. Numbering the two protons as particles 1 and 2 and the neutron as particle 3, the scattering state initiated by proton 1 scattering from a deuteron composed of nucleons 2 and 3 is given by

$$\Psi_1^{(+)} = i\epsilon G\Phi_1, \quad (\text{A1})$$

where G the full resolvent operator and

$$|\Phi_1\rangle = |\phi_d(23)\rangle |q_0\rangle_1. \quad (\text{A2})$$

The scattering state antisymmetrized in the two protons is then explicitly given by

$$\Psi^{(+)} = (1 - P_{12})\Psi_1^{(+)} \quad (\text{A3})$$

and enters the matrix element for elastic p-d scattering as

$$M = \langle \Phi_{q'} | (V_{12} + V_{13}) | \Psi^{(+)} \rangle = \langle \Phi_{q'} | (V_{12} + V_{13})(1 - P_{12}) | \Psi_1^{(+)} \rangle. \quad (\text{A4})$$

Here $|\Phi_{q'}\rangle$ is defined analogously to $|\Phi_1\rangle$. Attempts to derive Lippmann-Schwinger equations for the fully antisymmetrized scattering state given in Eq. (A3) did not lead to expressions, which are applicable in practice [21].

As an aside, we would like to mention a fact, which may not generally be known. The fully antisymmetrized state $\Psi^{(+)}$ obeys the same Lippmann Schwinger equation as $\Psi_1^{(+)}$, namely

$$\Psi_1^{(+)} = \Phi_1 + G_1(V_{13} + V_{12})\Psi_1^{(+)}. \quad (\text{A5})$$

The simple reason is that due to Lippmann identities [23], $\Psi_2^{(+)} = P_{12}\Psi_1^{+}$ obeys the homogeneous version of Eq. (A5). The same is true for a target composed of a general number of

A nucleons (N neutron and Z protons). This simply reflects the fact that Eq. (A5) does not uniquely define the scattering state. Using Eq. (A5) for the state $\Psi^{(+)}$ is only meaningful, if in the course of the solution antisymmetrization is always imposed, which is not easily implemented.

Here we do not want to follow these thoughts. Instead we apply the operator P_{12} in Eq. (A4) to the left. This results in

$$\begin{aligned} M &= \langle \Phi_{\mathbf{q}'} | (V_{12} + V_{13}) | \Psi_1^{(+)} \rangle - \langle \phi_{\mathbf{q}'} | (V_{12} + V_{23}) | \Psi_1^{(+)} \rangle \\ &= \langle \Phi_{\mathbf{q}'} | (1 - P_{12}) V_{12} | \Psi_1^{(+)} \rangle + \langle \Phi_{\mathbf{q}'} | V_{13} | \Psi_1^{(+)} \rangle - \langle \Phi_{\mathbf{q}'} | P_{12} V_{23} | \Psi_1^{(+)} \rangle. \end{aligned} \quad (\text{A6})$$

The generalization to a number of A target particles (N neutrons and Z protons) can be easily obtained in a similar fashion.

The first term in the second equality of Eq. (A6) imposes antisymmetry among the projectile target protons by antisymmetrizing the final channel state. The second term includes the proton-neutron interaction, which requires no antisymmetrization. The last term is an exchange term different from the first one, since the interaction V_{23} acts inside the initial target state. As a note, this last term is always considered of higher order in applications of the spectator expansion model [5], and is thus neglected when only the first order term is considered.

Since the physical interaction V_{12} between the two protons has to be represented under all circumstances in physically allowed states, i.e. states which are antisymmetric under exchanges of (12), the operation $(1 - P_{12})$ provides just a factor of 2. Neglecting the last term in Eq. (A6) we are left with

$$M \rightarrow 2 \langle \Phi_{\mathbf{q}'} | V_{12} | \Psi_1^{(+)} \rangle + \langle \Phi_{\mathbf{q}'} | V_{13} | \Psi_1^{(+)} \rangle. \quad (\text{A7})$$

In the following we sketch the steps usually carried out in the spectator expansion. The Green's operator G_1 in Eq. (A5) is projected onto a part propagating in the deuteron target state only, G_1^b , and a remainder

$$\Psi_1^{(+)} = \Phi_1 + G_1^b (V_{13} + V_{12}) \Psi_1^{(+)} + G_1^c (V_{13} + V_{12}) \Psi_1^{(+)}. \quad (\text{A8})$$

By introducing

$$\chi_{\mathbf{q}}^{(+)} = \Phi_{\mathbf{q}} + G_1^c (V_{13} + V_{12}) \chi_{\mathbf{q}}^{(+)}, \quad (\text{A9})$$

one finds together with $G_1^b = \int d^3 q' | \Phi_{\mathbf{q}'} \rangle g_0(\mathbf{q}') \langle \Phi_{\mathbf{q}} |$

$$\Psi_1^{(+)} = \chi_{\mathbf{q}_0}^{(+)} + \int d^3 q' \chi_{\mathbf{q}'}^{(+)} g_0(\mathbf{q}') \langle \Phi_{\mathbf{q}'} | (V_{13} + V_{12}) | \Psi_1^{(+)} \rangle. \quad (\text{A10})$$

In view of Eq. (A7) it appears to be inevitable to treat the actions of V_{12} and V_{13} separately and work with two coupled equations referring to the proton and neutron parts of M, respectively. However, doing so does not result in a single optical potential, which can serve as driving term for a one-body Lippmann-Schwinger equation for the elastic amplitude.

From Eq. (A10) one can derive two coupled equations for the matrix elements involving V_{12} and V_{13} :

$$\begin{aligned}
\langle \Phi_{\mathbf{q}} | V_{12} | \Psi_1^{(+)} \rangle &= \langle \Phi_{\mathbf{q}} | V_{12} | \chi_{\mathbf{q}_0}^{(+)} \rangle \\
&\quad + \int d^3 q' \langle \Phi_{\mathbf{q}} | V_{12} | \chi_{\mathbf{q}'}^{(+)} \rangle g_0(\mathbf{q}') \langle \Phi_{\mathbf{q}'} | (V_{12} + V_{13}) | \Psi_1^{(+)} \rangle \\
\langle \Phi_{\mathbf{q}} | V_{13} | \Psi_1^{(+)} \rangle &= \langle \Phi_{\mathbf{q}} | V_{13} | \chi_{\mathbf{q}_0}^{(+)} \rangle \\
&\quad + \int d^3 q' \langle \Phi_{\mathbf{q}} | V_{13} | \chi_{\mathbf{q}'}^{(+)} \rangle g_0(\mathbf{q}') \langle \Phi_{\mathbf{q}'} | (V_{12} + V_{13}) | \Psi_1^{(+)} \rangle
\end{aligned} \tag{A11}$$

The next step is to evaluate Eq. (A9). If one defines

$$V_{1i} \chi_{\mathbf{q}}^{(+)} \equiv T_i | \Phi_{\mathbf{q}} \rangle, \tag{A12}$$

where $i = 1, 2$, one obtains for Eq. (A9)

$$T_i \Phi_{\mathbf{q}} = V_{1i} \Phi_{\mathbf{q}} + V_{1i} G_1^c \sum_{j=2,3} T_j \Phi_{\mathbf{q}}. \tag{A13}$$

Collecting all $T_i \Phi_{\mathbf{q}}$'s on the left side and inverting the result yields

$$T_i \Phi_{\mathbf{q}} = \tau_i \Phi_{\mathbf{q}} + \tau_i G_1^c T_j \Phi_{\mathbf{q}} \quad (j \neq i), \tag{A14}$$

where the operators τ_i obey

$$\tau_i = V_{1i} + V_{1i} G_1^c \tau_i. \tag{A15}$$

This result corresponds to the familiar Watson expansion [1]. In first order one neglects the second part in Eq. (A14), which describes the consecutive scattering of the projectile proton on both target nucleons. In this first order approximation, the only one which has been applied in practice, one has

$$T_i \Phi_{\mathbf{q}} \rightarrow \tau_i \Phi_{\mathbf{q}}, \tag{A16}$$

and the optical potentials for pp and np scattering are given by

$$\begin{aligned}
\langle \Phi_{\mathbf{q}} | V_{12} | \chi_{\mathbf{q}'}^{(+)} \rangle &\rightarrow \langle \Phi_{\mathbf{q}} | \tau_2 | \Phi_{\mathbf{q}'} \rangle \\
\langle \Phi_{\mathbf{q}} | V_{13} | \chi_{\mathbf{q}'}^{(+)} \rangle &\rightarrow \langle \Phi_{\mathbf{q}} | \tau_3 | \Phi_{\mathbf{q}'} \rangle.
\end{aligned} \tag{A17}$$

Denoting the two transition amplitudes in Eq. (A11) for simplicity by $\bar{Z}_2 \equiv \langle \Phi_{\mathbf{q}} | V_{12} | \chi_{\mathbf{q}_0}^{(+)} \rangle$ and $\bar{Z}_3 \equiv \langle \Phi_{\mathbf{q}} | V_{13} | \chi_{\mathbf{q}_0}^{(+)} \rangle$ and using a matrix notation, indicated by the 'bar'-operators, Eq. (A11) takes the form

$$\begin{aligned}
\bar{Z}_2 &= \bar{\tau}_2 + \bar{\tau}_2 \bar{g}_0 (\bar{Z}_2 + \bar{Z}_3) \\
\bar{Z}_3 &= \bar{\tau}_3 + \bar{\tau}_3 \bar{g}_0 (\bar{Z}_2 + \bar{Z}_3).
\end{aligned} \tag{A18}$$

It remains to solve Eq. (A15), which takes the form

$$\tau_i = V_{1i} + V_{1i} G_1 \tau_i - V_{1i} G_1^b \tau_i. \tag{A19}$$

The propagator G_1 still contains the interaction of the struck target nucleon i with the other target nucleon. The simplest way to solve Eq. (A19) would be to replace

$$G_1 \rightarrow G_0. \quad (\text{A20})$$

As an aside, expanding $G_1 = G_0 + G_0 V_{23} G_1$ would lead to ‘medium corrections’ given by the potential V_{23} . However, for the clarity of presentation, we do not want to pursue this further. In the approximation introduced by Eq. (A20) and introducing the free NN t-matrix t_{1i} Eq. (A19) simplifies to

$$\tau_i = t_{1i} - t_{1i} G_1^b \tau_i. \quad (\text{A21})$$

Sandwiched between channel states this reads in matrix notation

$$\bar{\tau}_i = \bar{t}_{1i} - \bar{t}_{1i} \bar{g}_0 \bar{\tau}_{1i} = (1 + \bar{t}_{1i} \bar{g}_0)^{-1} \bar{t}_{1i}. \quad (\text{A22})$$

If one keeps the interaction V_{23} in G_1 one faces a three-body problem, which has never been solved correctly in that context. The work in [22] provides an approximate treatment of the interaction of the struck target nucleon with the remainder nucleons treated through a mean field interaction. With the three-body technology available today, a correct treatment appears to be feasible and worthwhile.

Continuing with the approximation of Eq. (A20) and inserting the expression of Eq. (A22) into Eqs. (A18) one finds

$$\begin{aligned} \bar{Z}_2 &= \bar{t}_{12} + \bar{t}_{12} \bar{g}_0 \bar{Z}_3 \\ \bar{Z}_3 &= \bar{t}_{13} + \bar{t}_{13} \bar{g}_0 \bar{Z}_2. \end{aligned} \quad (\text{A23})$$

After a partial wave decomposition, these are simple one-dimensional integral equations. Their on-shell solution provides according to Eq. (A7) the physical transition amplitude

$$M = 2\bar{Z}_2 + \bar{Z}_3. \quad (\text{A24})$$

The optical potentials occurring in Eqs. (A18)

$$\bar{\tau}_{2,3} \equiv \langle \Phi_{\mathbf{q}} | \tau_{2,3} | \Phi_{\mathbf{q}'} \rangle \quad (\text{A25})$$

are the full-folding expressions of the NN t-matrices modified according to Eq. (A21) and integrated over the single nucleon density matrix generated from the deuteron wave function. The NN t-matrix contains the free 3N propagator $G_0 = (E + i\varepsilon - \frac{p^2}{m} - \frac{3}{4m}q^2)$ and thus in addition to the kinetic energy of relative motion within the pair, $\frac{p^2}{m}$, also the kinetic energy of the pair as a whole in relation to the third particle, $\frac{3}{4m}q^2$. In Ref. [7] the kinetic energy of the pair is treated within the full-folding model, while in Refs. [6,8] this dependence is frozen into a constant. At this point we do not want to discuss the rationale for the two different treatments, but rather refer to the literature.

It is appealing to compare the expressions of Eq. (2.27) for the optical potentials in the Faddeev treatment to the one of Eq. (A22) in the spectator expansion. However, this is quite hampered. In Eq. (2.27) a NN t-matrix t_c occurs which is modified by a deuteron state for the same pair of nucleons as for the t-matrix, whereas in Eq. (A22) the deuteron is composed by a different pair of nucleons. Furthermore, in Eq. (2.27) there is the correction term $(1 - G_0 g_0^{-1})$ and the factor 2 resulting from $(1 - P_{1j})$. In the spectator expansion scheme the factor of 2 enters only in the final form of Eq. (A24) and not in the integral equations (A18), which are set up for distinguishable particles.

In view of all these differences a numerical study would be of great interest to clarify at least in the three-body context the validity of the spectator expansion against the systematic Faddeev approach.

APPENDIX B: THE TRANSITION POTENTIALS

The transition potentials entering the coupled integral equations for the p-³He and d-d channels are defined in Eq. (3.60). After considering the amplitudes ψ_i^c and ψ_i^d up to second order in T^c and \tilde{T}^c we find for the optical potential for elastic p-³He scattering, $V_{\mathbf{u}\mathbf{u}}$, the expression given in Eq. (3.65). In a similar fashion we obtain for the remaining transition potentials the following:

The d-d to d-d transition potential is given by

$$\begin{aligned} V_{\mathbf{v}\mathbf{v}} = & \langle \Phi_{\mathbf{v}\mathbf{v}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) G_0 T^c P | \Phi_{dd,\mathbf{v}}^F \rangle \\ & + \langle \Phi_{\mathbf{v}\mathbf{v}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) G_0 T^c P (-P_{34}) G_0 T^c P | \Phi_{dd,\mathbf{v}}^F \rangle \end{aligned} \quad (\text{B1})$$

The d-d to p-³He transition potential is given by

$$\begin{aligned} V_{\mathbf{u}\mathbf{v}} = & \langle \Phi_{\mathbf{u}\mathbf{v}} | V_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle + \langle \Phi_{\mathbf{u}\mathbf{v}} | V_{12} P (-P_{34}) G_0 T^c P | \Phi_{dd,\mathbf{v}}^F \rangle \\ & + \langle \Phi_{\mathbf{u}\mathbf{v}} | V_{12} P (-P_{34}) G_0 T^c P (-P_{34}) G_0 T^c P | \Phi_{dd,\mathbf{v}}^F \rangle \\ & + \langle \Phi_{\mathbf{u}\mathbf{v}} | V_{12} P G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) G_0 T^c P | \Phi_{dd,\mathbf{v}}^F \rangle \end{aligned} \quad (\text{B2})$$

The p-³He to d-d transition potential is given by

$$\begin{aligned} V_{\mathbf{v}\mathbf{u}} = & \langle \Phi_{\mathbf{v}\mathbf{u}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) | \Phi_{\mathbf{u}}^F \rangle + \langle \Phi_{\mathbf{v}\mathbf{u}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) G_0 T^c P (-P_{34}) | \Phi_{\mathbf{u}}^F \rangle \\ & + \langle \Phi_{\mathbf{v}\mathbf{u}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) G_0 T^c P (-P_{34}) G_0 T^c P (-P_{34}) | \Phi_{\mathbf{u}}^F \rangle \\ & + \langle \Phi_{\mathbf{v}\mathbf{u}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) G_0 T^c P G_0 \tilde{T}^c \tilde{P} (1 - P_{34}) | \Phi_{\mathbf{u}}^F \rangle \end{aligned} \quad (\text{B3})$$

After introducing for $G_0 T^c$ and $G_0 \tilde{T}^c$ the approximations given in Eqs. (3.72) and (3.73) as well as considering only terms up to second order in t_{12} , we obtain for the transition potentials:

$$\begin{aligned} V_{\mathbf{v}\mathbf{v}} = & \langle \Phi_{\mathbf{v}\mathbf{v}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) \Lambda G_0 t_{12} P + V_{12} \tilde{P} (1 - P_{34}) \Lambda G_0 t_{12} P G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \\ & + \langle \Phi_{\mathbf{v}\mathbf{v}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) \Lambda G_0 t_{12} P (-P_{34}) \Lambda G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle . \end{aligned} \quad (\text{B4})$$

$$\begin{aligned} V_{\mathbf{u}\mathbf{v}} = & \langle \Phi_{\mathbf{u}\mathbf{v}} | V_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \\ & - \langle \Phi_{\mathbf{u}\mathbf{v}} | V_{12} P P_{34} \Lambda G_0 t_{12} P + V_{12} P P_{34} \Lambda G_0 t_{12} P G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \\ & + \langle \Phi_{\mathbf{u}\mathbf{v}} | V_{12} P P_{34} \Lambda G_0 t_{12} P P_{34} \Lambda G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \\ & + \langle \Phi_{\mathbf{u}\mathbf{v}} | V_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} (1 - P_{34}) \Lambda G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \end{aligned} \quad (\text{B5})$$

and

$$\begin{aligned} V_{\mathbf{v}\mathbf{u}} = & \langle \Phi_{\mathbf{v}\mathbf{u}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) | \Phi_{\mathbf{u}}^F \rangle \\ & - \langle \Phi_{\mathbf{v}\mathbf{u}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) \Lambda G_0 t_{12} [1 + P G_0 t_{12}] P P_{34} | \Phi_{\mathbf{u}}^F \rangle \\ & + \langle \Phi_{\mathbf{v}\mathbf{u}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) \Lambda G_0 t_{12} P P_{34} \Lambda G_0 t_{12} P P_{34} | \Phi_{\mathbf{u}}^F \rangle \\ & + \langle \Phi_{\mathbf{v}\mathbf{u}}^{dd} | V_{12} \tilde{P} (1 - P_{34}) \Lambda G_0 t_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} (1 - P_{34}) | \Phi_{\mathbf{u}}^F \rangle . \end{aligned} \quad (\text{B6})$$

After the same simplifications leading to Eq. (3.75) we obtain the following expressions for the transition potentials:

$$\begin{aligned}
V_{\mathbf{v}'\mathbf{v}} = & \langle \Phi_{\mathbf{v}'}^{dd} | 2V_{12}\tilde{P}\Lambda G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \\
& + \langle \Phi_{\mathbf{v}'}^{dd} | 2V_{12}\tilde{P}\Lambda G_0 t_{12} P G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \\
& - \langle \Phi_{\mathbf{v}'}^{dd} | 2V_{12}\tilde{P}\Lambda G_0 t_{12} P P_{34}\Lambda G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle,
\end{aligned} \tag{B7}$$

$$\begin{aligned}
V_{\mathbf{u}'\mathbf{v}} = & \langle \Phi_{\mathbf{u}'} | V_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle - \langle \Phi_{\mathbf{u}'} | V_{12} P P_{34} \Lambda G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \\
& - \langle \Phi_{\mathbf{u}'} | V_{12} P P_{34} \Lambda G_0 t_{12} P [1 - P_{34}\Lambda] G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle \\
& + \langle \Phi_{\mathbf{u}'} | 2V_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} \Lambda G_0 t_{12} P | \Phi_{dd,\mathbf{v}}^F \rangle
\end{aligned} \tag{B8}$$

and

$$\begin{aligned}
V_{\mathbf{v}'\mathbf{u}} = & \langle \Phi_{\mathbf{v}'}^{dd} | 2V_{12}\tilde{P} | \Phi_{\mathbf{u}}^F \rangle - \langle \Phi_{\mathbf{v}'}^{dd} | 2V_{12}\tilde{P}\Lambda G_0 t_{12} P P_{34} | \Phi_{\mathbf{u}}^F \rangle \\
& - \langle \Phi_{\mathbf{v}'}^{dd} | 2V_{12}\tilde{P}\Lambda G_0 t_{12} P [1 - P_{34}\Lambda] G_0 t_{12} P P_{34} | \Phi_{\mathbf{u}}^F \rangle \\
& + \langle \Phi_{\mathbf{v}'}^{dd} | 4V_{12}\tilde{P}\Lambda G_0 t_{12} P \Lambda_{dd} G_0 t_{12} \tilde{P} | \Phi_{\mathbf{u}}^F \rangle.
\end{aligned} \tag{B9}$$

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FIGURES

FIG. 1. Born series for the four-nucleon breakup process $p\text{-}^3\text{He} \rightarrow \text{pppn}$. Particle 4 is singled out as projectile. For further explanation see text.

FIG. 2. The fully antisymmetrized four-nucleon breakup operator U_0 . Since the target state is assumed to be antisymmetric only the exchange terms with the projectile have to be considered. This leads to four terms with each nucleon being the projectile.

FIG. 3. Born series for the three-body fragmentation process $p\text{-}^3\text{He} \rightarrow \text{ppd}$. Particle 4 is singled out as projectile. For further explanation see text.

Fig. 1

The diagram illustrates an equation in a diagrammatic notation. On the left, four horizontal lines are labeled 1, 2, 3, and 4 from top to bottom. Lines 1, 2, and 3 enter a circle labeled $U_0^{(4)}$, and all four lines exit to the right, entering a semi-circular cap labeled ϕ . This is followed by an equals sign. To the right of the equals sign is a sum of terms. The first term consists of lines 1, 2, and 3 entering a semi-circular cap labeled ϕ , while line 4 has a single vertex connected to the cap by a vertical dashed line. The second term has two vertices on line 4, connected by a vertical dashed line, with the top vertex connected to the cap. The third term has three vertices on line 4, with the top vertex connected to the cap. This pattern continues with an ellipsis indicating further terms in the series. Each term in the sum is preceded by a plus sign.

$$\begin{aligned}
 & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \text{---} \bigcirc^{(4)} U_0 \text{---} \phi = \text{---} \phi + \text{---} \phi + \text{---} \phi \\
 & + \text{---} \phi + \text{---} \phi + \text{---} \phi + \dots
 \end{aligned}$$

Fig. 2

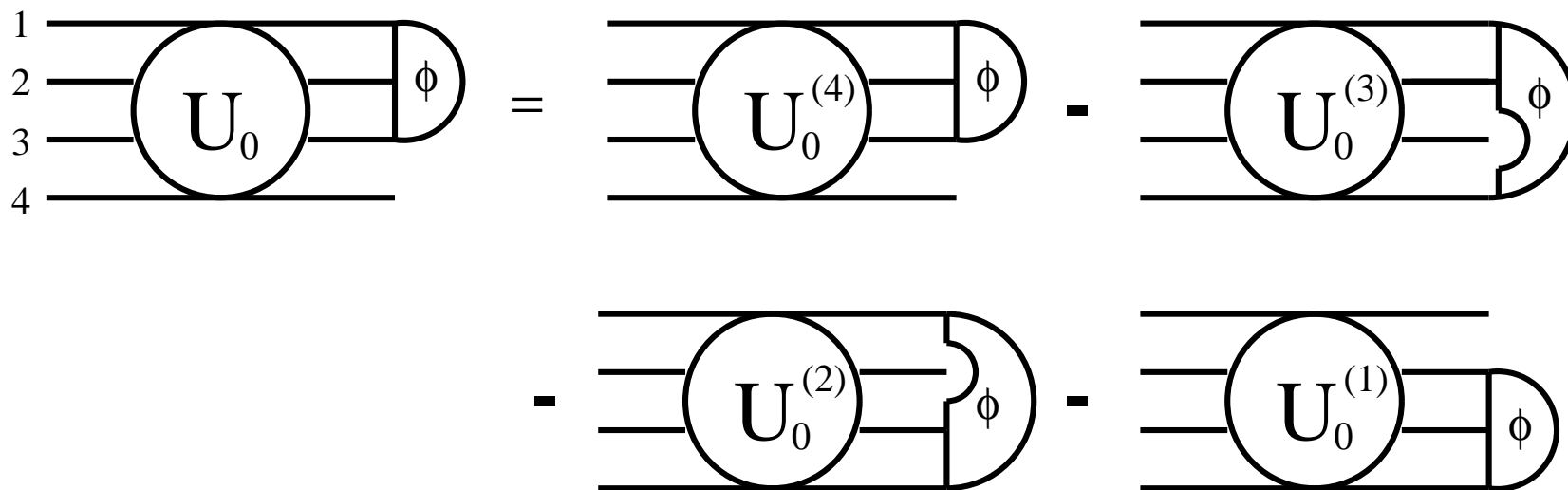


Fig. 3

